The exam is due by Wednesday, **3/23/25, at 3pm**, ETZ. I expect a legible, correctly ordered, written-on-one side only, stapled hard-copy turned in to me in person.

Legal things to consult: Your brain; the chapters we've covered in the book for the course; your homeworks; your class notes; the whiteboards; Mathematica, WolframAlpha, or similar **only to calculate cross products or as explicitly permitted by a problem**; anything else allowed by a particular problem; and, questions to me via email or in person. If you are unsure, please reach out to me. *Everything else is forbidden*, and violations will result in lowered grades and/or J-board hearings. In particular, this means NO consulting other people and NO use of ANYTHING online (besides what is allowed). Also, if I am convinced that even one person cheated, the final will be 'in-class' during the scheduled three hours. I don't need proof for this. I just need to believe that at least one person cheated.

Externalities: As usual, I will check email frequently and below are extra office hours for the exam. I reserve the right to set a point-price price-point to answer questions. By all means, ask any questions you may have; I'll let you know if it costs to get an answer. Moreover, for some problems, I may be willing to give a hint for points, but you should think of this as a last resort. Clarifications remain a bargain: Completely free!

Note: Each office hour visit is 5 minutes only. If there are others waiting, you may get back in line. If no one else is waiting, you may come back after 5 minutes have passed.

Finally, again, the exam must be given to me in person. No late exams will be accepted.

**Office Hours:**

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|---|---|
| M | 12 – 12:50, 1:30 – 2:20 |
| Tu | 9 – 9:50, 12 – 12:50, 2:30 – 3:20 |
| W | 9:30 – 10:20, 12 – 12:50, 2 – 2:50 |

Do your best! Show **all** your work! All problems are 20 points!

Please write your name on the BACK of your exam. Thank you!

Vectors and VVFs are written in **bold** font, as **u**, **v**, **w** and **r**(t), etc.

1. Find the equation of the plane in $\mathbb{R}^3$ containing the triangle with vertices {(6,0,0); (0,4,0); (0,0,2)} by first finding the general form as defined in the book. Then, write the equation for the plane as a scalar field

$$f: \mathbb{R}^2 \to \mathbb{R}.$$

No computer assistance, except (optionally) to graph the plane to check your equation.

2. For t in $[0, 2\pi]$, let $\mathbf{r}_2(t) = \langle 3\cos(t), 2\sin(t) \rangle$. (What is the range/image/trace of $\mathbf{r}_2(t)$?)

- (5 points) Find $\mathbf{T}(t)$.
- (10 points) Find by hand the general equation for the curvature $\boldsymbol{\kappa}$ of $\mathbf{r}_2(t)$.
- (5 points) Find the curvature for $t = \pi/2$ and $t = \pi$.

3. Define/find $\mathbf{r}_3(t)$ by 'lifting' the trace/image of $\mathbf{r}_2(t)$ up to the plane from problem 1. That is,

- (5 points) Use $\mathbf{r}_2(t)$ and work from problem 1 to find $\mathbf{r}_3(t) = \langle x(t), y(t), z(t) \rangle$.
- (10 points) Deriving everything by hand, except (optionally) for the cross product, find the general equation for the curvature $\boldsymbol{\kappa}$ of $\mathbf{r}_3(t)$. The cross product may be found by hand, but computer assistance is allowed.
- (5 points) Find the curvature for $t = \pi/2$ and $t = \pi$.

4. What is the area of the region R in $\mathbb{R}^2$ enclosed by $\mathbf{r}_2(t)$?

5. What is the surface area of the region D in $\mathbb{R}^3$ enclosed by $\mathbf{r}_3(t)$ in the plane from problem 1?