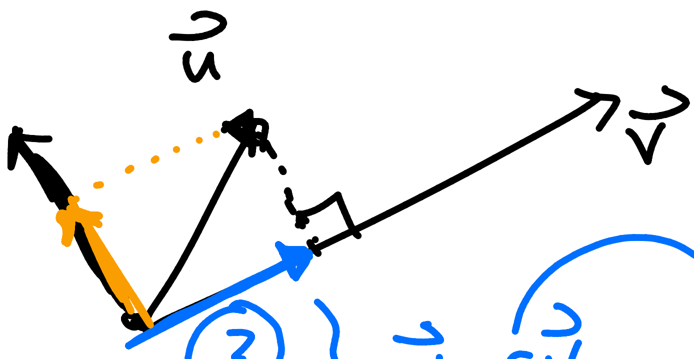
Multi Vector Review



② $\vec{w} = c\vec{v}$ how to find c??

$$\text{proj}_{\vec{v}}\vec{u} = \vec{w} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v}$$

proj. of $\vec{u}$ onto $\vec{v}$.

c

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dot prods — closest to vector mult. $\vec{u}\cdot\vec{v}=0 \iff \vec{u}\perp\vec{v}$

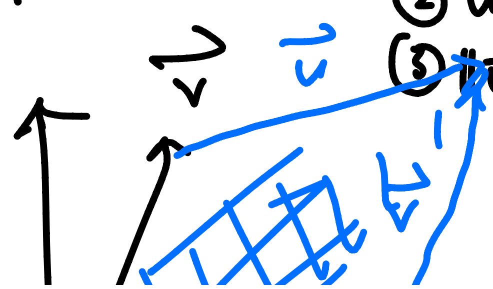
cross product — $\times : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$

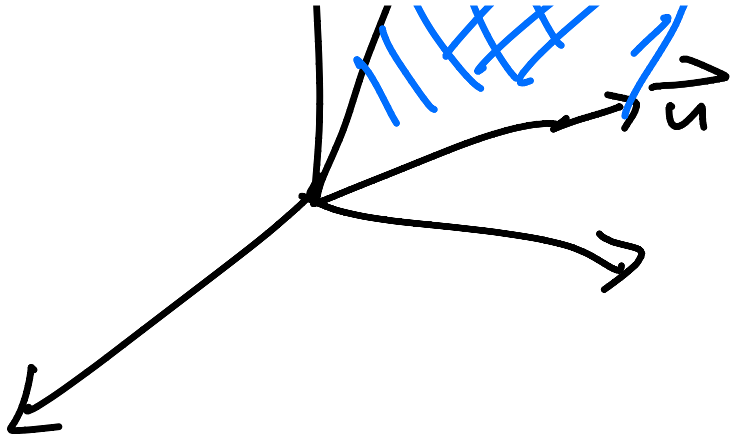
$\vec{u},\vec{v} \in \mathbb{R}^3$ $\qquad \vec{u}\times\vec{v}$

① $\vec{u}\times\vec{v} \perp \vec{u},\vec{v}$

② $\vec{u}\times\vec{v}$ points uniquely

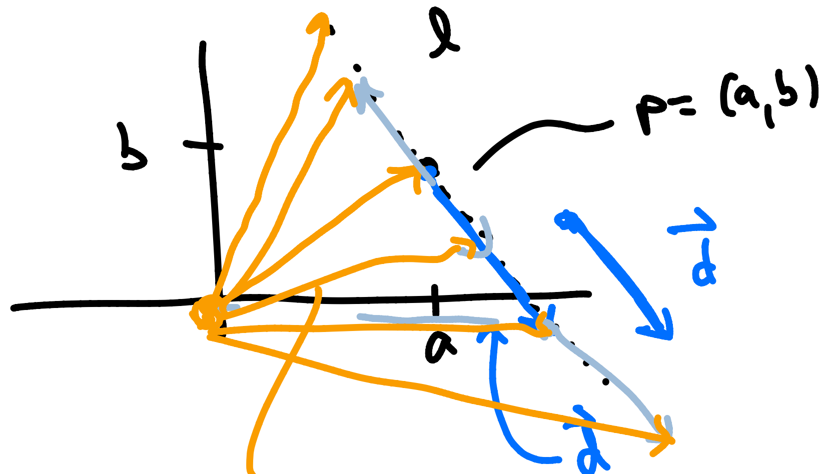
③ $\|\vec{u}\times\vec{v}\|$ = area the parallelogram formed by $\vec{u},\vec{v}$.





(4) ONLY in $\mathbb{R}^3$.

<u>Lines</u>



$\vec{p} = \langle a, b \rangle$

<u>Note</u>: $\vec{p} + \vec{d}$ is on the line!

then: $\ell(t) = \vec{p} + t\vec{d}$

t parameter — exp. contr. invert.

Given: ℓ, pt. p on ℓ, $\vec{d} \parallel$ line : eq. is trivial!!

$$\ell(t) = \vec{p} + t\vec{d} = \langle p_1, p_2, p_3 \rangle + t\langle a, b, c \rangle$$

$= \langle p_1 + at,\ p_2 + bt,\ p_3 + ct \rangle$

parametric form

$\vec{p}$

Planes — find eq. for plane!

$P = (p_1, p_2, p_3)$

$\vec{n}$ – normal $\equiv \perp$ to every vector parallel to the plane

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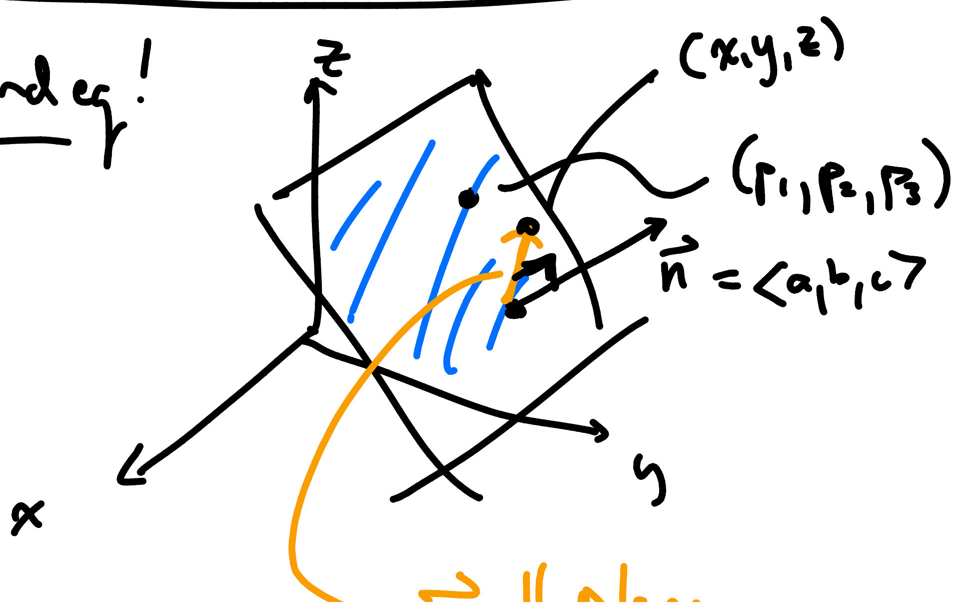
Find eq!



$\vec{v} \parallel$ plane

$\Rightarrow \vec{n} \cdot \vec{v} = 0$

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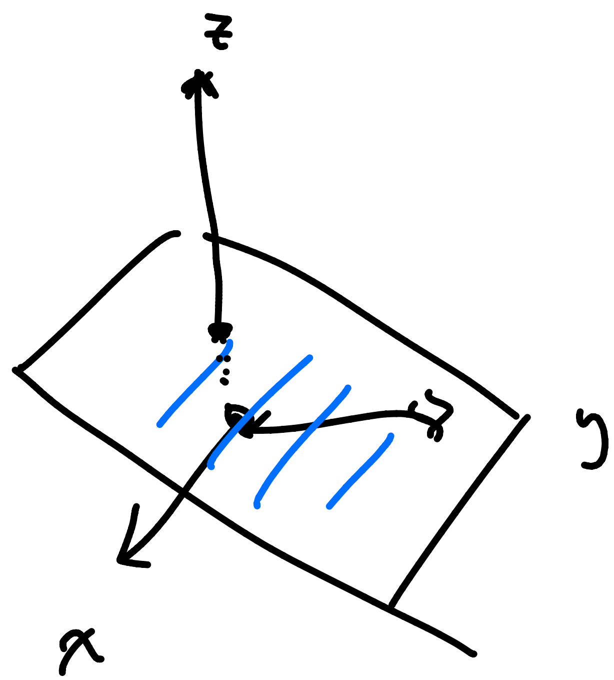
① "Move $\vec{p}$ to origin"

$\langle x-p_1, y-p_2, z-p_3 \rangle$, where $\langle x, y, z \rangle$ is a point in the plane.

$\langle x, y, z \rangle - \vec{p}$



② $\vec{n} \perp$ to all vectors in plane

$\vec{n} \perp$ to all vectors in translated plane

$\langle x-p_1, y-p_2, z-p_3 \rangle$

$\left[\langle x, y, z \rangle - \vec{p} \right.$

So:

③ $\vec{n} \cdot \langle x - p_1, y - p_2, z - p_3 \rangle = 0$

$\vec{n} \cdot \left[\langle x, y, z \rangle - \vec{p} \right] = 0$

$\Leftrightarrow \vec{n} \cdot \langle x, y, z \rangle - \vec{n} \cdot \vec{p} = 0$

$\Leftrightarrow \boxed{\vec{n} \cdot \langle x, y, z \rangle = \vec{n} \cdot \vec{p}}$

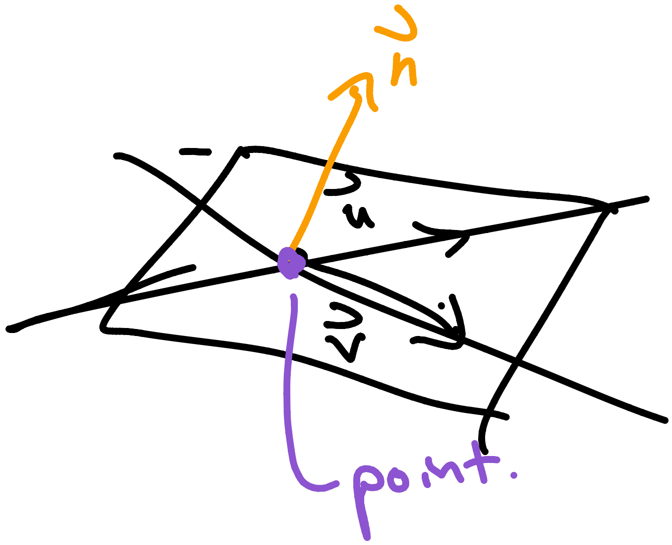
unknown vectorized point in plane.

that is: $\langle a, b, c \rangle \cdot \langle x, y, z \rangle = \langle a, b, c \rangle \cdot \langle p_1, p_2, p_3 \rangle$

$\Leftrightarrow \boxed{ax + by + cz = \underbrace{ap_1 + bp_2 + cp_3}_{\text{number}}}$

$a, b, c \in \mathbb{R}$

Eq. for a plane!

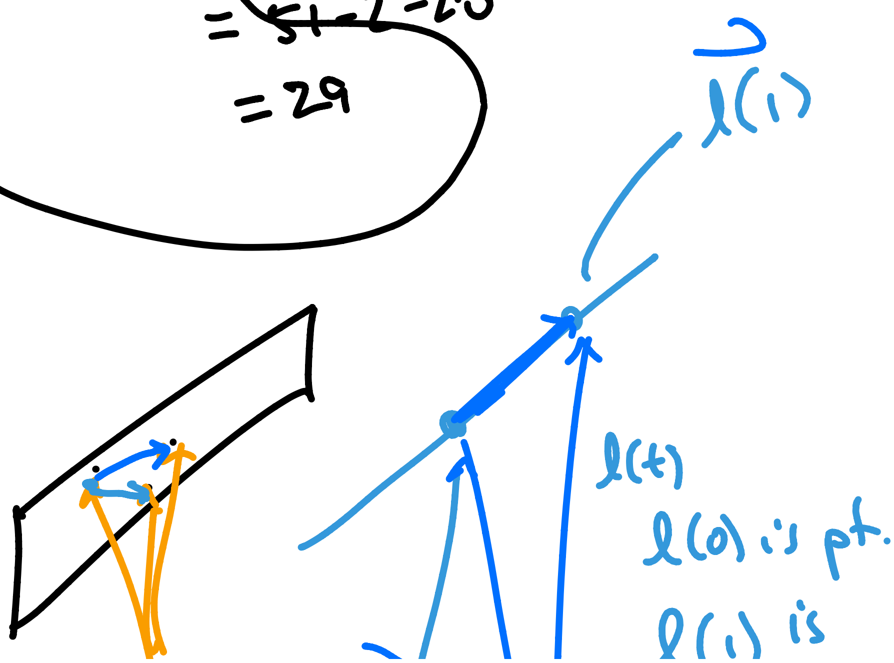


$\vec{n} = \langle 17, 2, -4 \rangle$

$p = (3, -1, 5)$

eq. for plane:

$$17x + 2y - 4z = (17 \cdot 3 + (2)(-1) + 5(-4))$$
$$= 51 - 2 - 20$$
$$= 29$$



$\vec{l}(0)$

another pt.

$l(0)$ is pt.

$\vec{l}(1) - \vec{l}(0)$ is a vector