Calculus I

3.3

IV = AV at (at least) one pt.

MVT

$\exists c \in (a,b)$

s.t $f'(c) = \dfrac{f(b)-f(a)}{b-a} = \dfrac{\Delta y}{\Delta x}$

IV = AV $\equiv$ slope

optimized / sketched derivatives

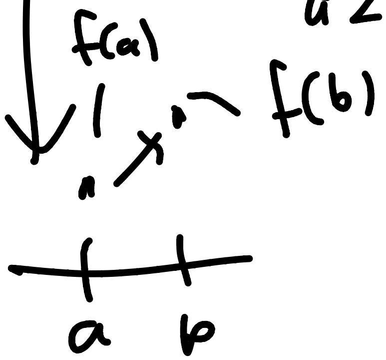
Increasing & Decreasing Fns.

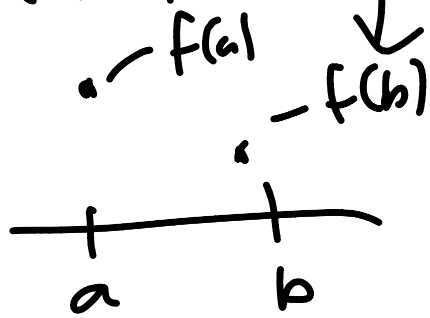
How does f' speak to these

?

$\boxed{\text{DEF}}$ f is <u>incr.</u> on I (interval), if $\forall a, b$ in I, $a < b$, $f(a) < f(b)$.

$\boxed{\text{DEF}}$ f is <u>decr.</u> on I, if $\forall a, b$ in I,
$a < b \Rightarrow f(a) > f(b)$





Thm Let $f : [a,b] \to \mathbb{R}$ be <u>nice</u>

$\left[\begin{array}{l} f \text{ is cont. on } [a,b] \\ f \text{ is diff. on } (a,b) \end{array}\right.$ THEN

$\boxed{f \text{ is}}$:

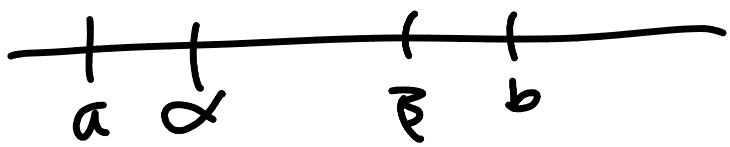
1. $f' > 0$ on (a,b) $\Rightarrow$ <u>increasing</u>
2. $f' < 0$ on (a,b) $\Rightarrow$ <u>decr.</u>
3. $f' \equiv 0$ on (a,b) $\Rightarrow$ constant

$$0 = f'(c) = \frac{f(b)-f(a)}{b-a} = 0$$

$c \in (a,b)$ $\Rightarrow f(b)-f(a)=0$

$\Rightarrow f(b)=f(a)$

proof by ~~MVT~~



$$0 = \frac{f(\beta)-f(\alpha)}{\beta-\alpha} = f'(c), \quad c \in (\alpha,\beta)$$

MVT

$$\Rightarrow f(\beta) = f(\alpha)$$

Let $\alpha, \beta \in (a,b)$

MVT:

$\exists c \in (\alpha,\beta)$

s.t.

$$f'(c) = \frac{f(\beta)-f(\alpha)}{\beta-\alpha} > 0$$

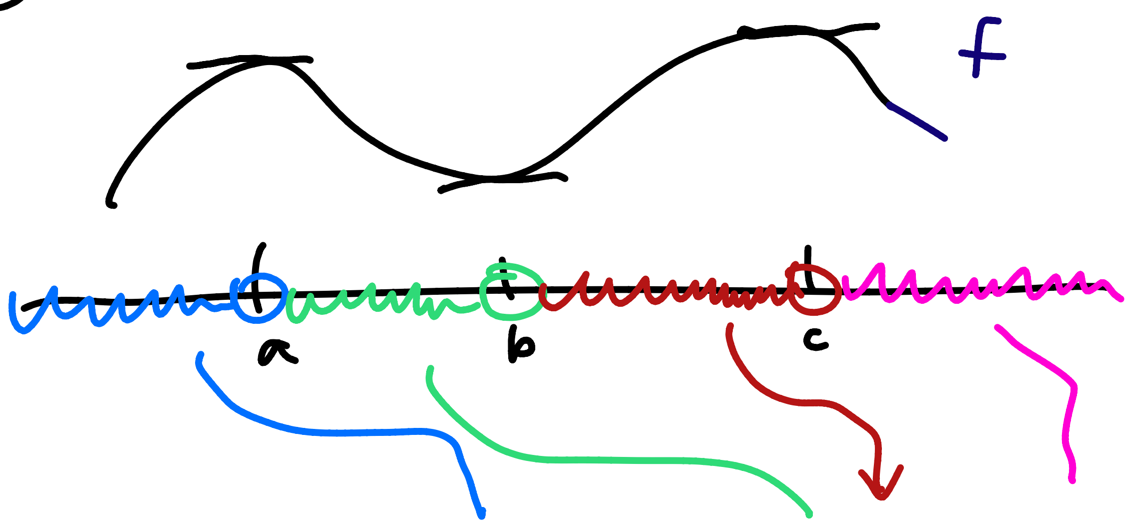
IV AV

$$\Rightarrow f(\beta) - f(\alpha) = \underbrace{\underbrace{(\beta-\alpha)}_{>0}\underbrace{f'(c)}_{>0}}_{>0}$$

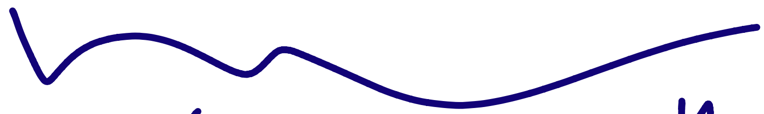
$$\Rightarrow \boxed{f(\beta) > f(\alpha)}$$

Protocol to use this thm:
(f is nice)

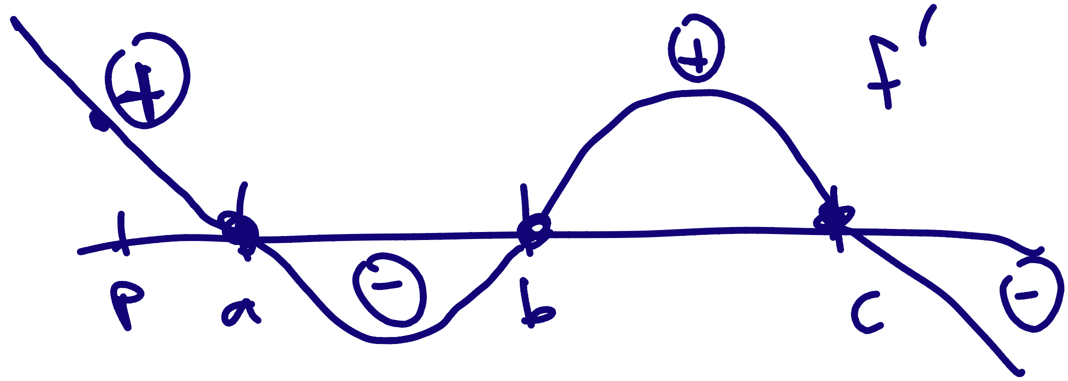
① Find criticals : $f'(x)=0$ (EVT)



(2) Then, $f' : \overset{+}{(-\infty, a)} \cup \overset{-}{(a, b)} \cup \overset{+}{(b, c)} \cup \underset{-}{(c, \infty)}$



f' is NEVER 0 on this.



(3) Choose any p in the non-zero interval, find $f'(p)$, then incr. on whole int. if $f'(p) > 0$.
otherwise, decr.

Pg. 150

11 ish $f(x)=x^3-5x^2+7x-1$

(a) $f'(x)=3x^2-10x+7$

(b) find incr. decr. intervals

(i) $f'(x)=0$:

$$3x^2-10x+7=0$$

(ii)

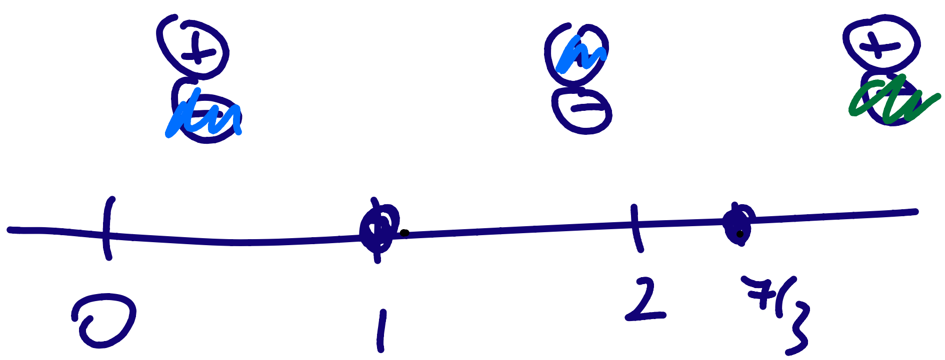
$$(3x-7)(x-1)=0$$

$\therefore\ x=1$ or $3x-7=0$

$\Rightarrow 3x=7$

$\Rightarrow x=7/3$

f'



(ii) $3(0)^2-10(0)+7=7$

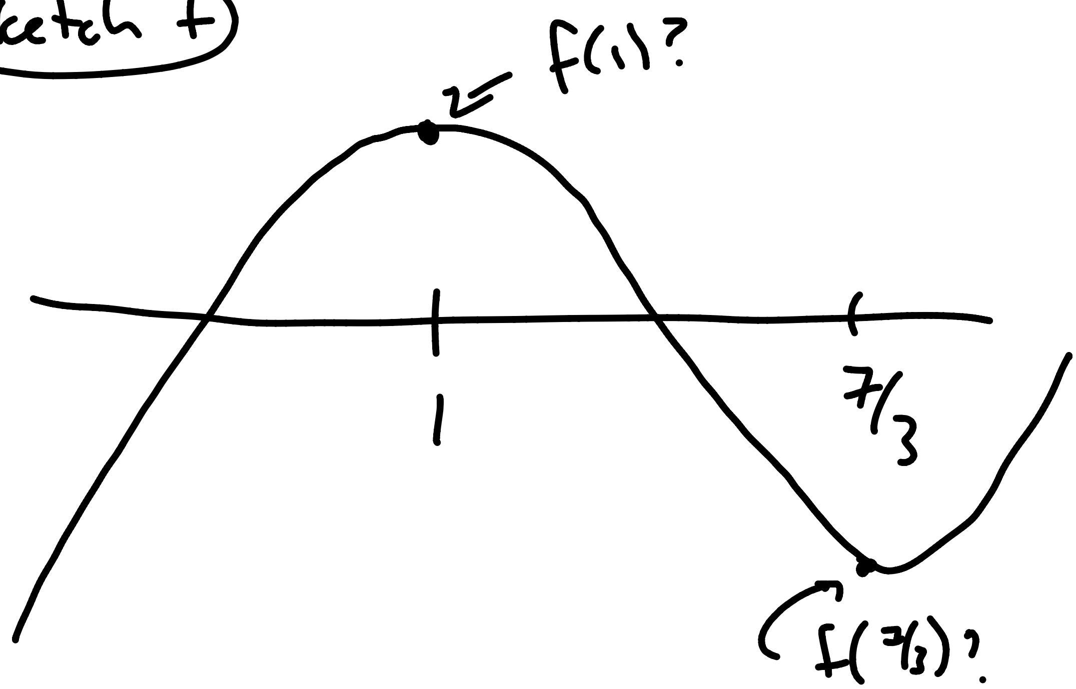
$3(2)^2-10(2)+7=-1$

$12-20+7$

$3(100)^2 - 10(100) + 7 =$

$30{,}000 - 1{,}000 + 7 = 29{,}007$

sketch f



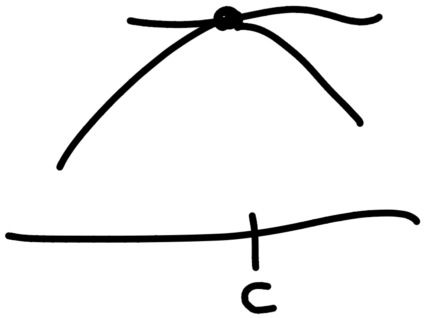
First Derivative Test

$\boxed{\text{Thm}}$ Let f be nice, $f'(c) = 0$. If

$f'(x) > 0$, $x < c$ AND

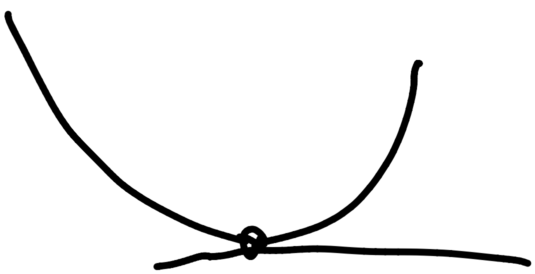
$f'(x) < 0$, $x > c$, THEN

f is a local max.



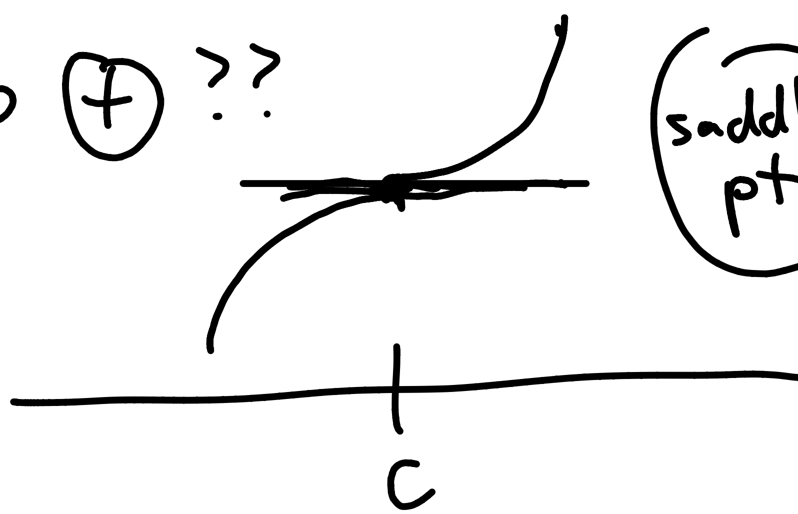
similar: $\quad$ c

$- \quad 0 \quad +$



local min!

$+ \quad 0 \quad +$??



saddle pt.

$$\left.\begin{aligned} f(x) &= x^3 \\ f'(x) &= 3x^2 \end{aligned}\right\} \text{criticals}$$

$$f' = 0$$

$$3x^2 = 0$$

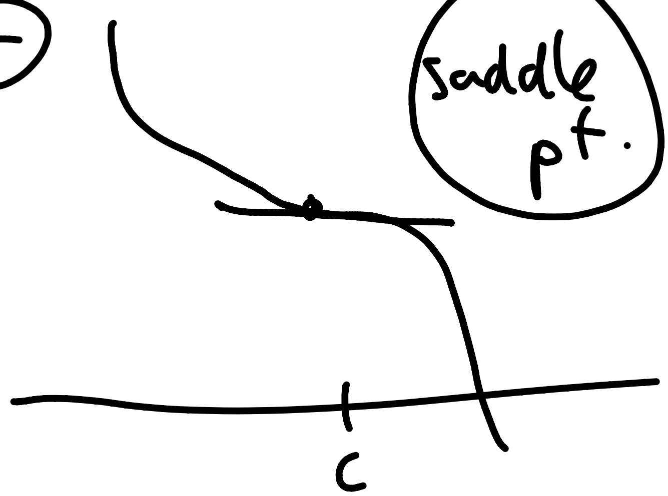
$$\Rightarrow x^2 = 0$$

$$\Rightarrow x = 0$$

$$f'(-1) = 3(-1)^2 = 3 \quad +$$

$$f'(1) = 3(1)^2 = 3 \quad +$$

$-$ 0 $-$



saddle pt.