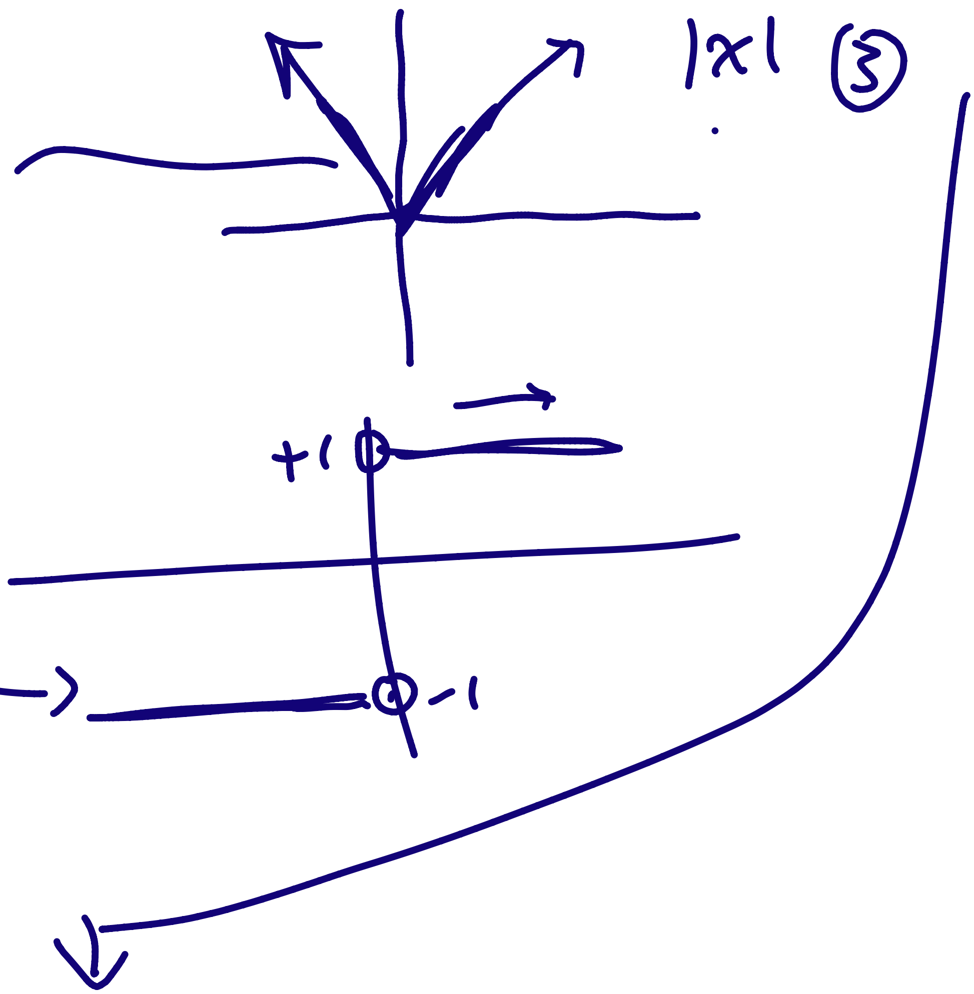
# Calc I

## I) Reminder: Extreme Value Thm

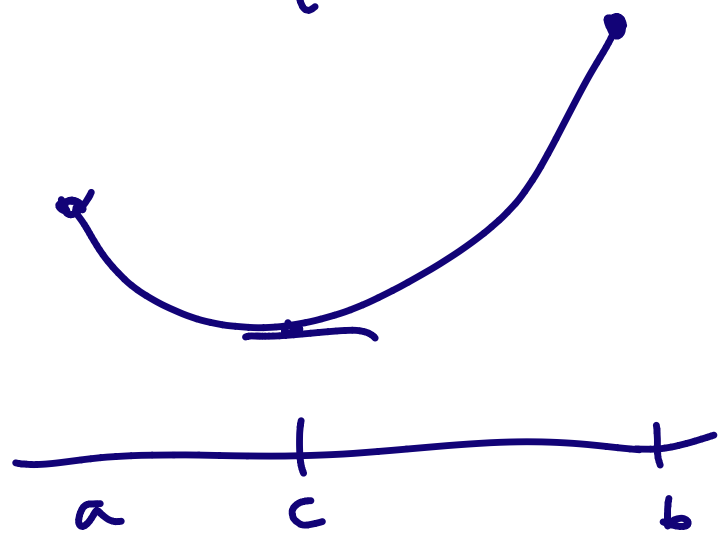
If f is cont. on [a,b] and c is

a ~~local~~ / global { max / min , then ① $f'(c)=0$

② $f'(c)$ doesn't exist



At an endpt: $\begin{cases} f(a) \\ f(b) \end{cases}$

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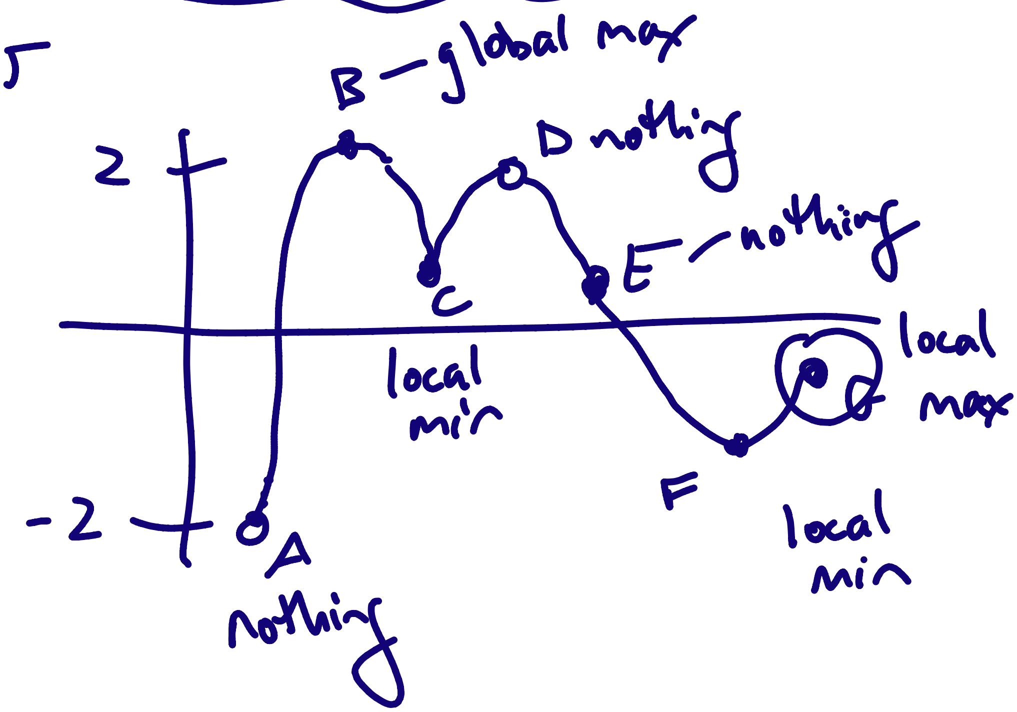


21 $f(x) = x + 3/x = x + 3x^{-1}$ on $[1,5]$

$f'(x) =$

$1 - 3x^{-2}$

$= 1 - \frac{3}{x^2}$

① $f'(c)$ doesn't exist?

Nope!

② $f'(c) = 0$

Solving!

$$1 - \frac{3}{x^2} = 0$$

$$\Rightarrow 1 = \frac{3}{x^2}$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3}$$

$-\sqrt{3} \notin [1,5]$

$\sqrt{3} \in [1,5]$

$\sqrt{3}$ is a candidate for max or min!

$f(1)$, $f(5)$, $f(\sqrt{3})$

$1 + 3/1 = 4$ $\quad 5 + 3/5 = 5\frac{3}{5}$ $\quad \sqrt{3} + 3/\sqrt{3}$

$\sqrt{3} + \sqrt{3} = 2\sqrt{3}$

smaller than 4!

$\frac{3}{\sqrt{3}} = \frac{(\sqrt{3})(\sqrt{3})}{\cancel{\sqrt{3}}}$

$f(\sqrt{3}) = 2\sqrt{3}$ is global min

$f(5) = 5^{3/5}$ is global max

## $\boxed{3.2}$

## Mean Value Theorem

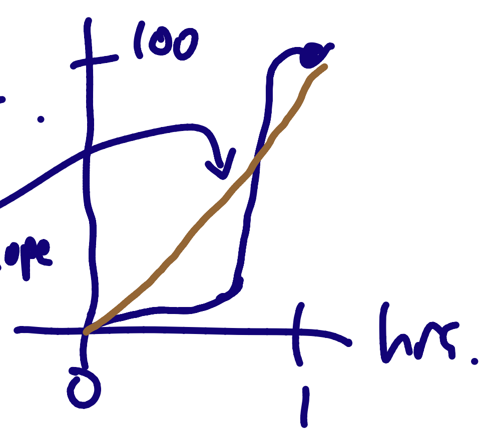
$f : [a,b] \to \mathbb{R}$ is cont.

$f : (a,b) \to \mathbb{R}$ is diff.

$$AV = \frac{f(b) - f(a)}{b-a}$$

$$= \frac{\Delta y}{\Delta x}$$

slope



IV at $x=c$ is $f'(c)$

$\exists\, c \in (a,b)$ s.t.

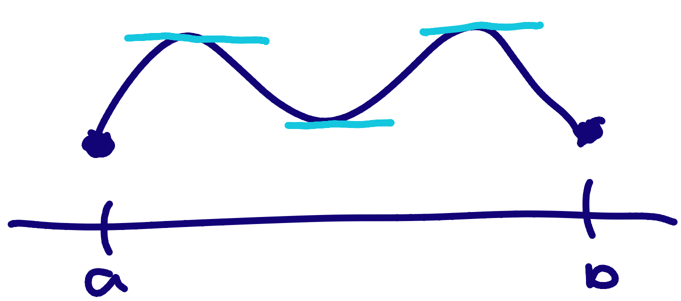
$$\underset{\text{IV}}{f'(c)} = \underset{\text{AV}}{\frac{f(b)-f(a)}{b-a}}$$

$\boxed{\text{PROOF}}$ Rolle's Thm

Let $f:[a,b]\to\mathbb{R}$ be cont.

$f:(a,b)\to\mathbb{R}$ be diff

$f(a)=f(b)$



Then, $\exists c \in (a,b)$ s.t.

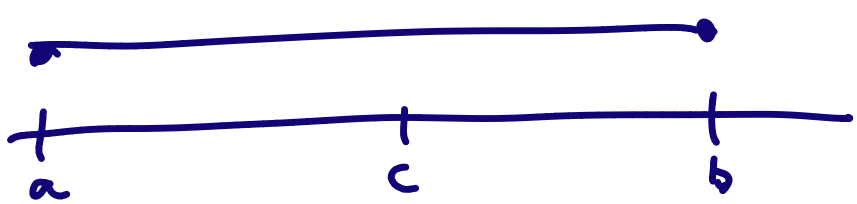
$$\underbrace{f'(c)}_{IV} = \underbrace{\frac{f(b)-f(a)}{b-a}}_{AV} = \frac{0}{b-a} = 0$$

**proof of Rolle's Thm:**

(2 cases)

(i) $f$ is constant: $\forall c \in [a,b]$,

$$f(c) = f(a) = f(b)$$




$\therefore f'(c) = 0$

(ii) $f$ is <u>not</u> constant. Then, by the EVT says MUST be both a global $\begin{cases} \max \\ \min \end{cases}$. WLOG (without loss of generality) suppose $f(a)$ is the min. Then, max is at $c \in (a,b)$.

$\therefore f'(c) = 0$

## Proof (cont.) of MVT

AV

$\boxed{\text{Define}}$ $g(x) = f(x) - \left[\dfrac{f(b) - f(a)}{b-a}\right] x$

(show $g(x)$ satisfies Rolle's thm)

(use conclusion from Rolle's to get concl. for MVT)

NTS: $g(a) = g(b) \iff \boxed{g(b) - g(a) = 0}$

need to show

$$g(b) - g(a) = \underbrace{\left[\cancel{f(b)} - \left[\frac{f(b)-f(a)}{b-a}\right] b\right]}_{g(b)} - \underbrace{\left[\cancel{f(a)} - \left[\frac{f(b)-f(a)}{b-a}\right] a\right]}_{g(a)}$$

$$f(b) - f(a) - \left[\frac{f(b)-f(a)}{b-a}\right] b + \left[\frac{f(b)-f(a)}{b-a}\right] a$$

$$= f(b) - f(a) + \left[\frac{f(b)-f(a)}{b-a}\right](a-b)(-1)(-1)$$

$$= f(b) - f(a) - \left[\frac{f(b)-f(a)}{b-a}\right](b-a)$$

$\therefore$ by Rolle's Thm: $\exists c \in (a,b)$ s.t.

$$g'(c) = 0$$

$$g(x) = f(x) - \left[\frac{f(b) - f(a)}{b - a}\right] x$$

$$\therefore\ g'(x) = f'(x) - \left[\frac{f(b) - f(a)}{b - a}\right] =$$

$$\therefore\ g'(c) = f'(c) - \left[\frac{f(b) - f(a)}{b - a}\right] = 0$$

$$\Rightarrow\ f'(c) = \frac{f(b) - f(a)}{b - a}$$

IV AV