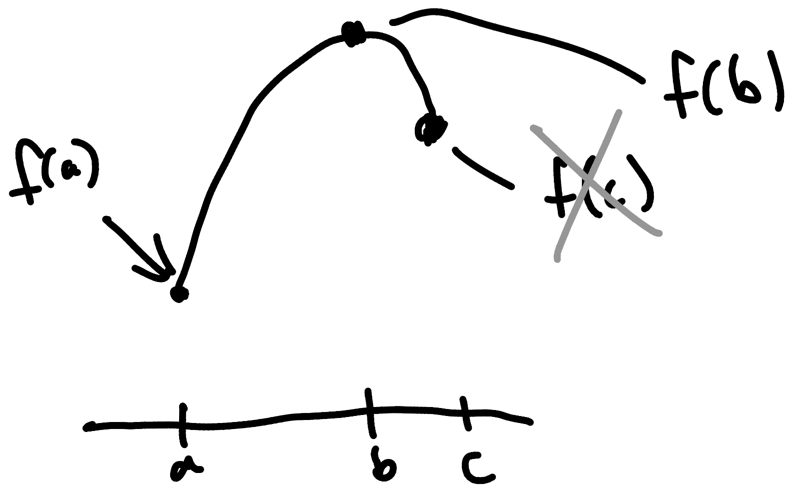
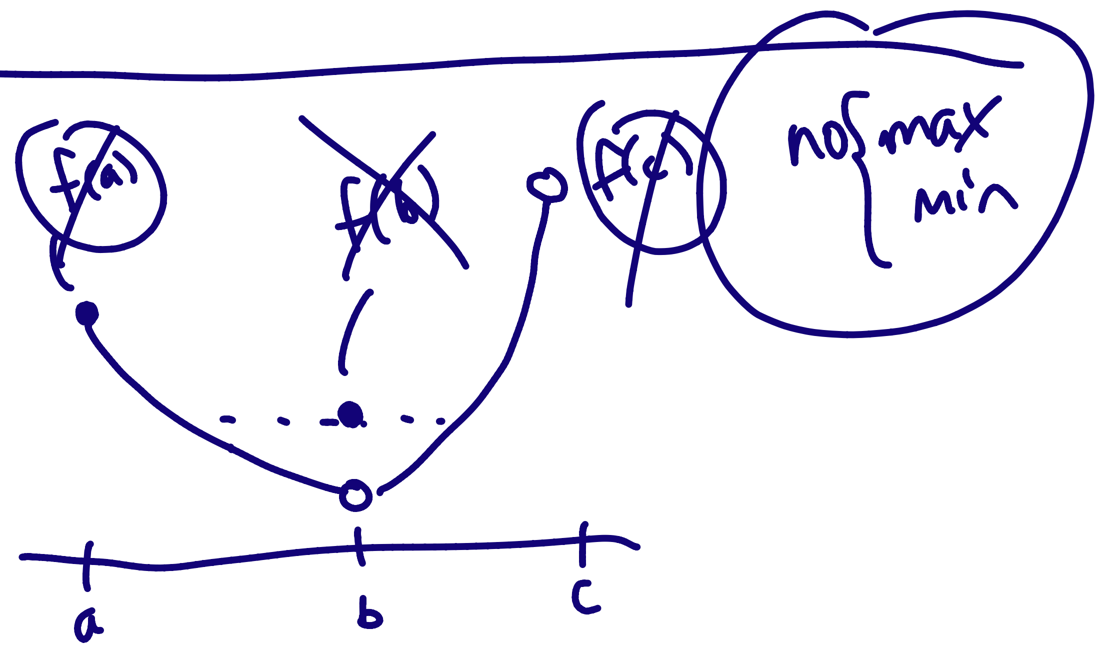
$\boxed{\text{Calc I}}$

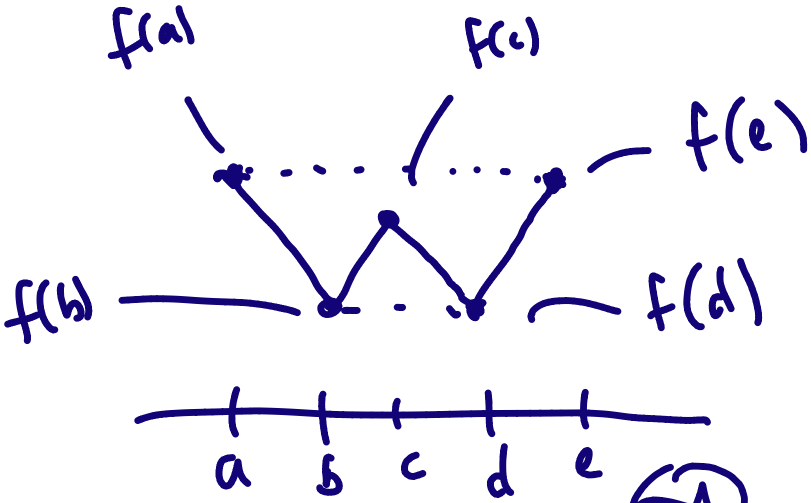
Extreme Values X-Val

biggest (max)

smallest (min)







max val: $f(a), f(e)$

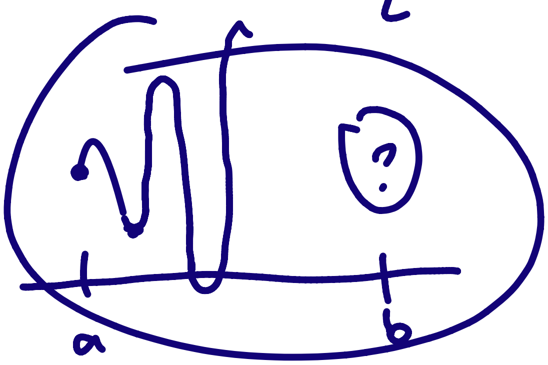
min val: $f(b), f(d)$

$\boxed{\text{Thm}}$ Let $f: I \to \mathbb{R}$ be continuous on $[a,b]$. Then

I: interval $[a,b]$, (a,b), $[a,b)$, $(a,b]$

f has both a $\begin{cases}\text{max} \\ \text{min}\end{cases}$.

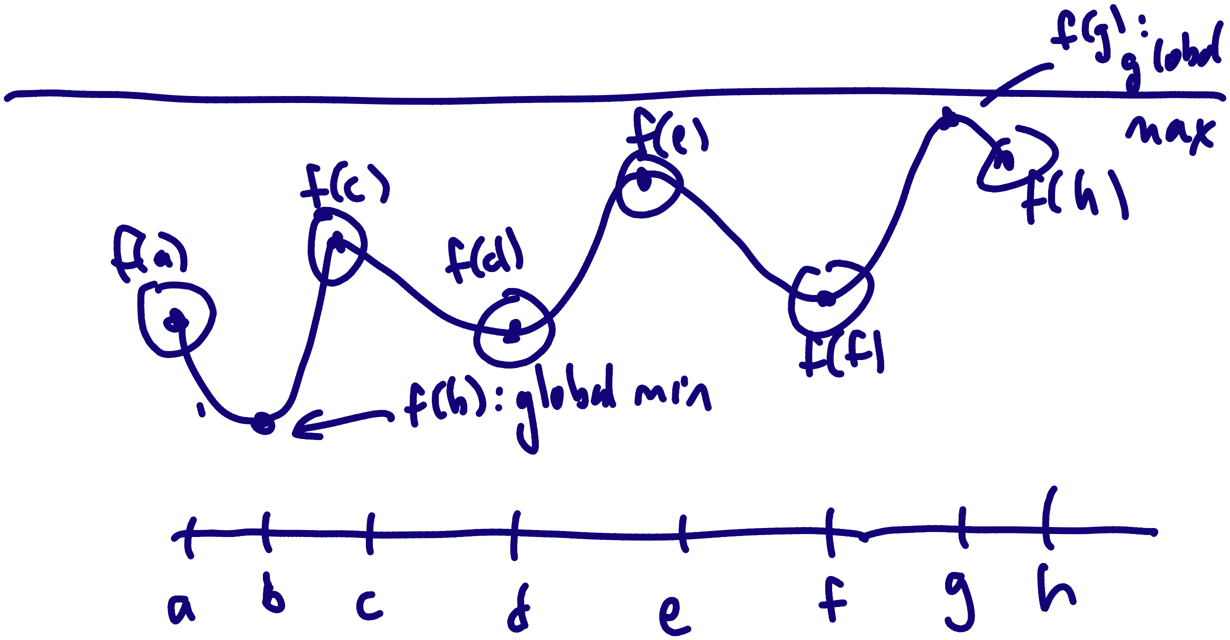
$\Big\{$ global max, min



$\boxed{\text{DEF}}$ global $\begin{cases} \text{max} \\ \text{min} \end{cases}$

max: $\exists c \in I$ s.t. $f(c) \geq f(x)$ $\forall x \in I$

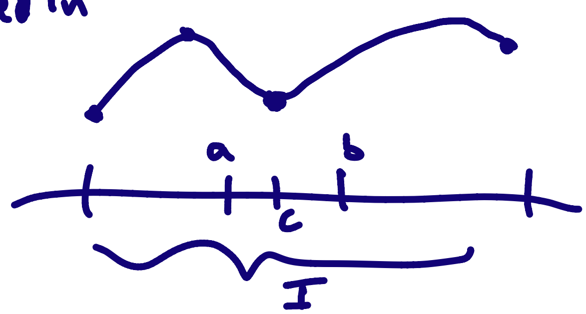
min: $\exists d \in I$ s.t. $f(d) \leq f(x)$ $\forall x \in I$



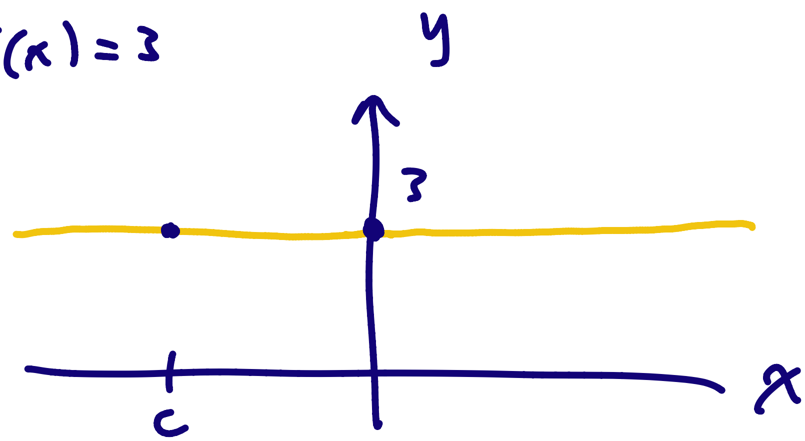
$\boxed{\text{DEF}}$ Local $\begin{cases} \text{max} \\ \text{min} \end{cases}$: $c \in I$, $f(c)$ is a local $\begin{cases} \text{max} \\ \text{or} \\ \text{min} \end{cases}$

if $c \in (a,b) \subset I$ and $\begin{array}{l} f(c) \geq f(x),\ x \in (a,b) \\ f(c) \leq f(x),\ x \in (a,b) \end{array}$

"contained in"



NOTE $f(x) = 3$



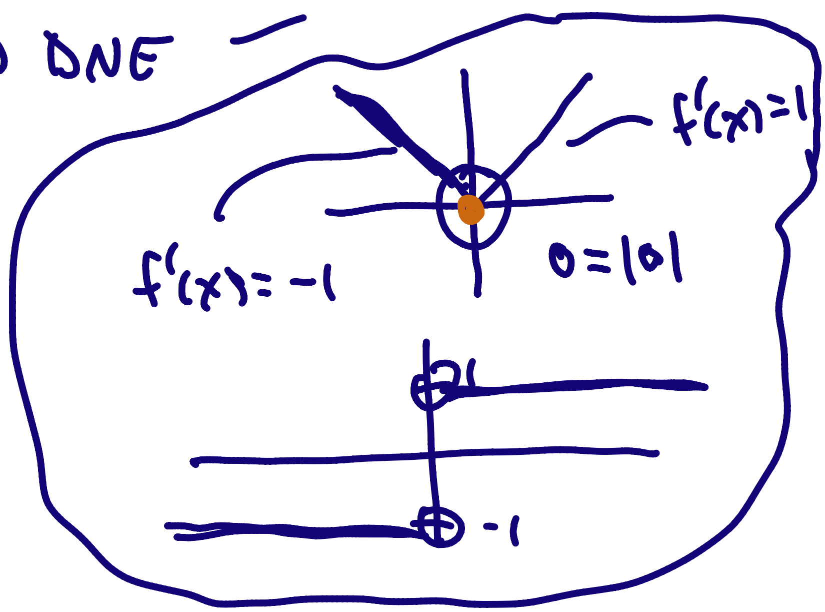
$f(c) \geq f(x)$ $\forall c$, $f(c)$ is a global $\begin{cases} \text{max} \\ \text{max} \end{cases}$

$f(c) \leq f(x)$

$\boxed{\text{DEF}}$ Let $f(c)$ exist. Then c is a <u>critical val</u>, if

1. $f'(c) = 0$
2. $f'(c)$ DNE

$f(x) = |x|$



DEF If c is critical val, then $(c, f(c))$ is a **critical point**.

THM Let $f:(a,b) \to \mathbb{R}$. Let $c \in (a,b)$ and $f(c)$ is a local $\left\{\begin{matrix} \text{max} \\ \text{min} \end{matrix}\right.$ at $(c, f(c))$.

Then, c is a **critical val**.

EX $f(x) = x^3$. Then $f'(x) = 3x^2$

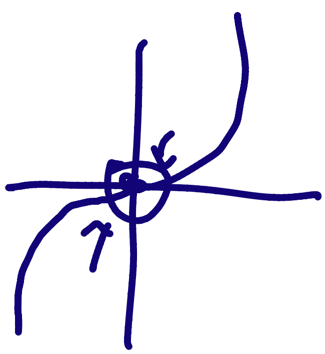
Set: $f'(x) = 0$

$\Rightarrow 3x^2 = 0$

$\Rightarrow x^2 = 0$

$\Rightarrow \boxed{x = 0}$

0 is a crit val.



NOT any kind of max or min.

$\boxed{\text{Algorithm}}$ Finding Extrema for $f : [a,b] \to \mathbb{R}$

① Find $f(a), f(b)$ (endpoint vals)

② Find crit vals ⓐ $f' = 0$
ⓑ f' DNE

③ Let $\{c_1, c_2, \cdots, c_n\}$ be crit. vals.
check $\{f(c_1), f(c_2), \ldots, f(c_n)\}$

④ Global $\left\{ \begin{matrix} \text{max} \\ \text{min} \end{matrix} \right.$ MUST occur at
(absolute) either endpts. or crit vals,
so check: $\{f(a), f(b), f(c_1), \cdots, f(c_n)\}$

$\boxed{\text{EX}}$ ㉓ pg. 135 $f(x) = e^x \cos x$ on $[0, \pi]$. Find global max, min:

① $f(0) = e^0 \cos(0) = 1 \cdot 1 = 1$
$f(\pi) = e^\pi \cos(\pi) = e^\pi(-1) = -e^\pi \sim -27$

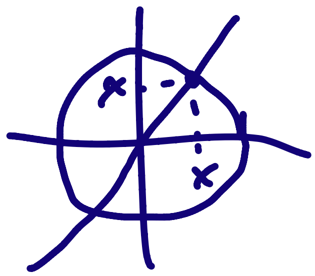
② $f'(x) = 0$: $f'(x) = e^x \cos x - (\sin x) e^x$
$= e^x(\cos x - \sin x)$

$f' = 0$:

$e^x(\cos x - \sin x) = 0$

$\Rightarrow \quad \cos x - \sin x = 0$ (b/c e^x is never 0)

$\Rightarrow \quad \cos x = \sin x$



$(x^2 + x^2) = 1^2$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

crit val — $x = \frac{1}{\sqrt{2}}$

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