Practice Test I Linear Algebra

1. Given the matrices *A* and *B*:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

a. (5 points) What is the domain for *AB* ? What is the codomain for *AB* ?

b. (5 points) What is the domain for *BA* ? What is the codomain for *BA* ?

c. (15 points) Is *AB* invertible? If it is, find $(AB)^{-1}$. If *AB* isn't invertible, explain (prove) why it isn't.

d. (15 points) Is *BA* invertible? If it is, find $(BA)^{-1}$. If *BA* isn't invertible, explain (prove) why it isn't.

2. Consider the linear transformation *T* as seen below. Answer the questions that follow with sufficient, convincing detail.

$$T((x, y, z)) = (x + z, y, -y + z)$$

a. (10 points) Find the standard matrix *A* associated with *T*.

b. (10 points) Is *A* injective? Either prove that it is, or find two vectors $\vec{u} \neq \vec{v}$ such that $A\vec{u} = A\vec{v}$.

c. (10 points) Is *A* surjective? Either prove that it is, or find a vector $\vec{b}$ such that there does not exist a vector $\vec{x}$ with $A\vec{x} = \vec{b}$.

d. (10 points) If *A* is both injective and surjective, find A^{-1}. If *A* isn't bijective, explain (prove) why it isn't.

3. (25 points) Set up, **but do not multiply out**, all of the matrices, including the data matrix D, required to rotate a unit square π/49 radians* in the counter-clockwise direction around the point (3, 3). The vertices of the square are at (3, 3), (4, 3), (4, 4), and (3, 4).

* I don't care about the exact numbers for the trig; just write the appropriate formulae.